SAMPLING

IMPORTANT TERMS

POPULATION / UNIVERSE

- All items ,elements , or observations of interest having similar properties are known as population .
- **It may be defined as the aggregate of all the units under consideration .**
- **The number of units belonging to a population is known as population size.**
- **Example : All the lamps produced by "General Electricals" in the past, present and future constitute the population**

POPULATION / UNIVERSE

Finite Population

- If a population comprises only a finite number of units, then it is known as a finite population.
- Population of students enrolled for CA Course , population of students in a class ,population of workers in factory .

Infinite Population

- If the population contains an infinite or uncountable number of units, then it is known as an infinite population.
- population of stars, the population of mosquitoes in Kolkata, the population of flowers in Mumbai, the population of insects in Delhi etc. are infinite population.

POPULATION / UNIVERSE

Existent Population

- A population consisting of real objects is known as an existent population.
- The population of the lamps produced by General Electricals

Hypothetical Population

- A population that exists just hypothetically like the population of heads when a coin is tossed infinitely is known as a hypothetical or an imaginary population

IMPORTANT TERMS

Census

- Study of every elements of population is called census .

SAMPLE

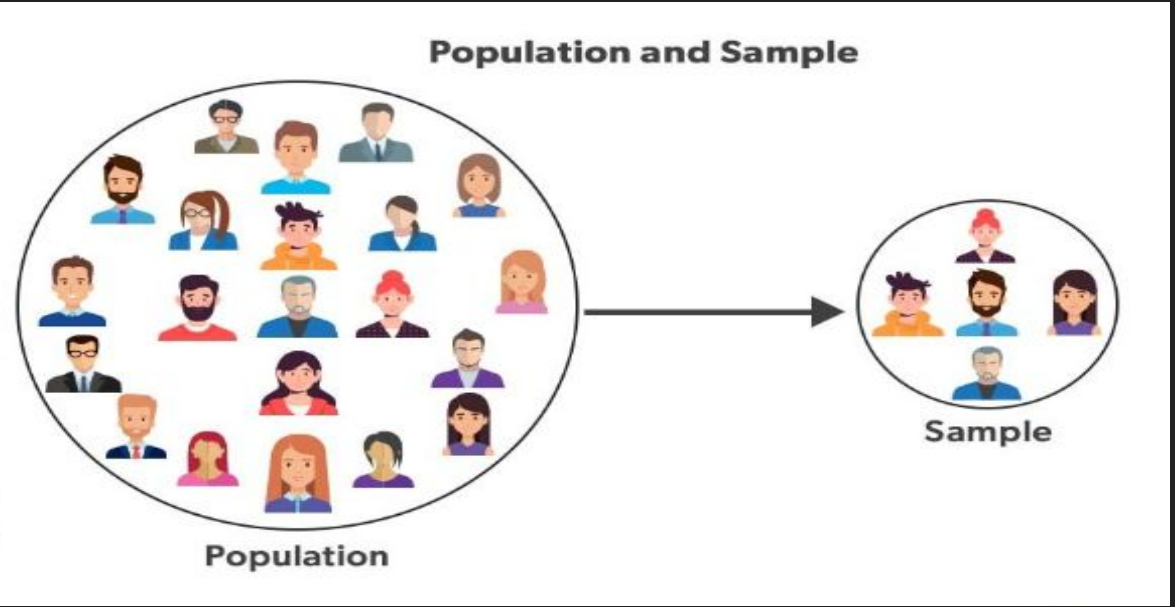
A representative subset of a population.

- A sample may be defined as a part of a population so selected with a view to representing the population in all its characteristics .
- selection of a proper representative sample is pretty important because statistical inferences about the population are drawn only on the basis of the sample observations.
- If a sample contains n units, then n is known as sample size.

SAMPLE

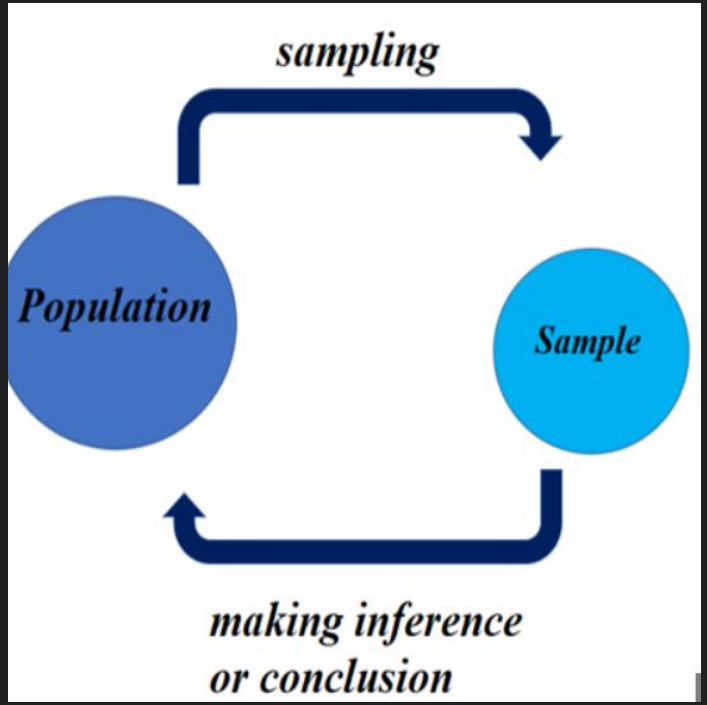
A representative subset of a population.

- The units forming the sample are known as “Sampling Units”.
- A detailed and complete list of all the sampling units is known as a “Sampling Frame”.
- **Example :**
- If a sample of 500 electrical lamps is taken from the production process of General Electricals.
- n = 500
- Sampling units = Electrical lamp


Population and Sample
Population
Sample

- Sampling is a technique of selecting individual members or subset of the population to make statistical inferences from them and estimate characteristics of the whole universe .

- EXAMPLE:
  A drug manufacturer would like to research the adverse side effects of a drug on the the country's population it is almost impossible to conduct a research study that involves everyone . In this case , the researcher decides a sample of people and then researches them .



RANDOM SAMPLING

- When each member of the population has an equal chance to belong to the sample, then sampling scheme is known as Simple Random Sampling .
- **Random sampling is done in two ways :**
- *SIMPLE RANDOM SAMPLING WITH REPLACEMENT*
- *SIMPLE RANDOM SAMPLING WITHOUT REPLACEMENT*

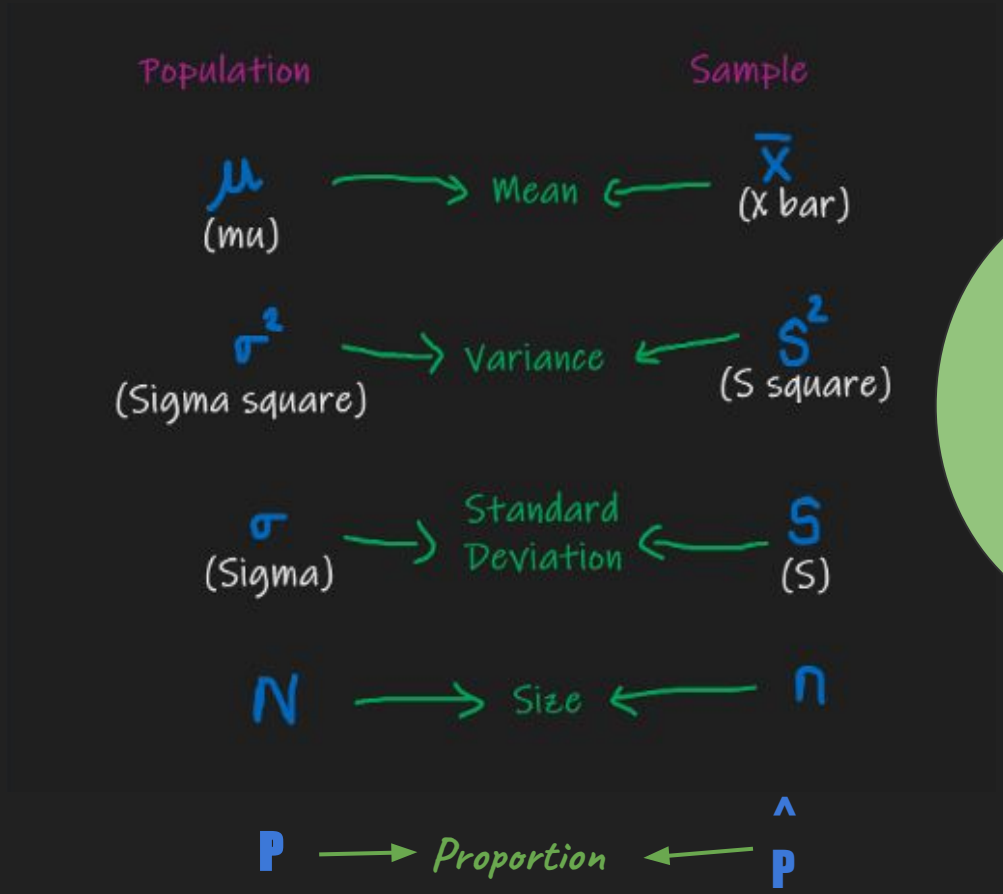
PARAMETER

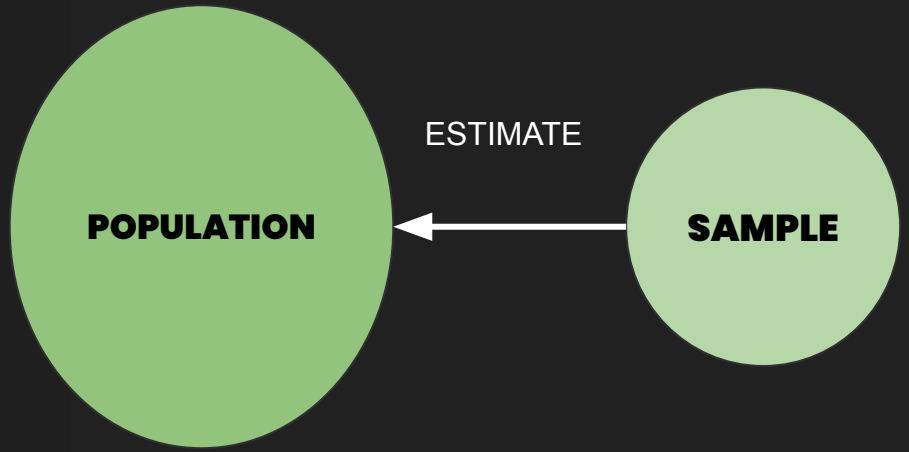
- A parameter may be defined as a characteristic of a population based on all the units of the population

STATISTICS

- A statistic may be defined as a statistical measure of sample observation and as such it is a function of sample observations. If the sample observations are denoted by $x_1, x_2, x_3, \ldots\ldots\ldots x_n$, then a statistic T may be expressed as $T = f(x_1, x_2, x_3, \ldots\ldots\ldots x_n)$
- A statistic is used to estimate a particular population parameter

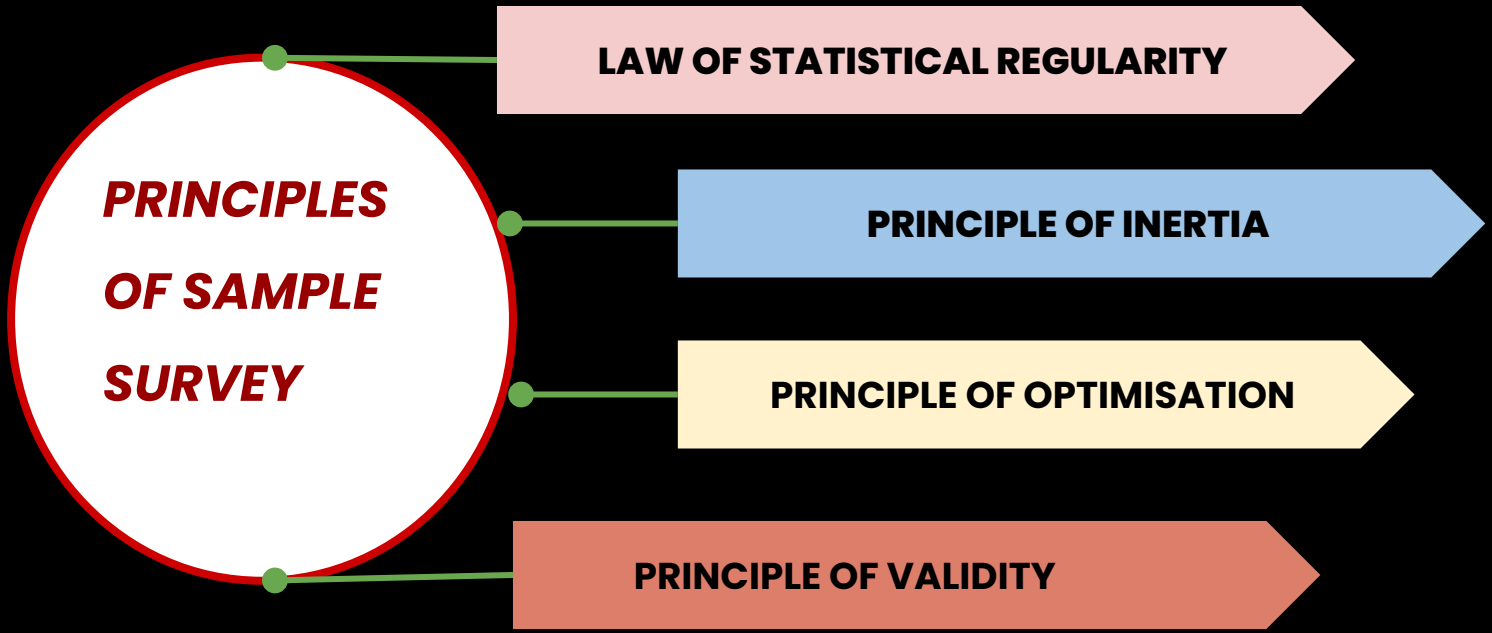
STATISTICAL CONSTANT OF POPULATION IS PARAMETER AND OF SAMPLE STATISTIC





- *A statistic is used to estimate a particular population parameter*

- Sample Survey is the study of the unknown population on the basis of a proper representative sample drawn from it .



LAW OF STATISTICAL REGULARITY

- According to the law of statistical regularity, if a sample of fairly large size is drawn from the population under discussion at random, then on an average the sample would possess the characteristics of that population.
- This principle emphasises on two factors

1. **Sample Size should be large :** As the sample size increases it becomes more and more representative of parent population and shows its characteristics .
2. **Sample must be drawn randomly :** Equal probability of being selected in the sample

PRINCIPLE OF INERTIA

- According to the principle of inertia of large numbers, The results derived from a sample size are likely to be more reliable, accurate and precise as the sample size increases, provided other factors are kept constant.

- This is a direct consequence of the first principle.

PRINCIPLE OF OPTIMISATION

- The principle of optimisation in sampling refers to the idea of selecting an efficient and effective sampling design to get the most accurate and reliable results while utilising resources efficiently .
- The principle of optimization ensures that an optimum level of efficiency at a minimum cost or the maximum efficiency at a given level of cost can be achieved with the selection of an appropriate sampling design.

PRINCIPLE OF VALIDITY

- The principle of validity states that a sampling design is valid only if it is possible to obtain valid estimates and valid tests about population parameters.
- Only a probability sampling ensures this validity.

COMPARISON BETWEEN SAMPLE SURVEY AND COMPLETE ENUMERATION

- When complete information is collected for all the units belonging to a population, it is defined as complete enumeration or census.

- In most cases, we prefer sample survey to complete enumeration due to the following factors:

Speed: As compared to census, a sample survey could be conducted, usually, much more quickly simply because in sample survey, only a part of the vast population is enumerated.

COMPARISON BETWEEN SAMPLE SURVEY AND COMPLETE ENUMERATION

Cost : **The cost of collection of data on each unit in case of sample survey is likely to be more as compared to census because better trained personnel are employed for conducting a sample survey.**

But when it comes to total cost, sample survey is likely to be less expensive as only some selected units are considered in a sample survey.

COMPARISON BETWEEN SAMPLE SURVEY AND COMPLETE ENUMERATION

Reliability : The data collected in a sample survey are likely to be more reliable than that in a complete enumeration because of trained enumerators better supervision and application of modern technique.

COMPARISON BETWEEN SAMPLE SURVEY AND COMPLETE ENUMERATION

Accuracy : **Every sampling is subjected to what is known as sampling fluctuation which is termed as sampling error.**

It is obvious that complete enumeration is totally free from this sampling error.

It may be noted that in sample survey, the sampling error can be reduced to a great extent by taking several steps like increasing the sample size, adhering to a probability sampling design strictly and so on.

COMPARISON BETWEEN SAMPLE SURVEY AND COMPLETE ENUMERATION

Necessity : **Sometimes, sampling becomes necessity. When it comes to destructive sampling where the items get exhausted like testing the length of life of electrical bulbs or sampling from a hypothetical population like coin tossing, there is no alternative to sample survey.**

However, when it is necessary to get detailed information about each and every item constituting the population, we go for complete enumeration.

ERRORS IN SAMPLE SURVEY

- Errors or biases in a survey may be defined as the deviation between the value of population parameter as obtained from a sample and its observed value.

- Errors are of two types.

  I. Sampling Errors

  II. Non-Sampling Errors

SAMPLING ERRORS

- Since only a part of population is investigated in sampling , every sampling design is subjected to this type of errors .
- **Factors contributing to sampling errors are as follows :**

  Errors arising out due to defective sampling design:
  Selection of a proper sampling design plays a crucial role in sampling. If a non- probabilistic sampling design is followed, the bias or prejudice of the sampler affects the sampling technique thereby resulting some kind of error.

SAMPLING ERRORS

- **Factors contributing to sampling errors are as follows :**
- ***Errors arising out due to substitution:*** **A very common practice among the enumerators is to replace a sampling unit by a suitable unit in accordance with their convenience when difficulty arises in getting information from the originally selected unit. Since the sampling design is not strictly adhered to, this results in some type of bias.**

SAMPLING ERRORS

- **Factors contributing to sampling errors are as follows :**
- **Errors owing to faulty demarcation of units:** It has its origin in faulty demarcation of sampling units. In case of an agricultural survey, the sampler has, usually, a tendency to underestimate or overestimate the character under consideration.

SAMPLING ERRORS

- **Factors contributing to sampling errors are as follows :**
- **Errors owing to wrong choice of statistics :** One must be careful in selecting the proper statistic while estimating a population characteristic.

- **Variability in the population:** Errors may occur due to variability among population units beyond a degree. This could be reduced by following somewhat complicated sampling design like stratified sampling,

NON - SAMPLING ERRORS

- Errors due to recording observations, biases on the part of the enumerators, wrong and faulty interpretation of data is termed as non-sampling errors.
  - This type of errors happen both in sampling and complete enumeration .
  - Some factors responsible for this particular kind of biases are preference for certain digits, ignorance, psychological factors , non- responses on the part of the interviewees wrong measurements of the sampling units, communication gap between the interviewers and the interviewees, incomplete coverage etc. on the part of the enumerators also lead to non-sampling errors.

SAMPLING DISTRIBUTION AND STANDARD ERROR OF STATISTIC

- Starting with a population of N units, we can draw many a sample of a fixed size n.
- In case of **sampling with replacement**, the **total number of samples** that can be drawn is N^n
- When it comes to **sampling without replacement** , the **total number of samples** that can be drawn is ${}^{N}C_{n}$

SAMPLING DISTRIBUTION AND STANDARD ERROR OF STATISTIC

- If we compute the value of a statistic, say mean, it is quite natural that the value of the sample mean may vary from sample to sample as the sampling units of one sample may be different from that of another sample.
- The variation in the values of a statistic is termed as “Sampling Fluctuations”.

SAMPLING DISTRIBUTION AND STANDARD ERROR OF STATISTIC

- If it is possible to obtain the values of a statistic from all the possible samples of a fixed sample size along with the corresponding probabilities, then we can arrange the values of the statistic, which is to be treated as a random variable, in the form of a probability distribution. Such a probability distribution is known as the sampling distribution of the statistic.

- The mean of the statistic, as obtained from its sampling distribution, is known as "Expectation" and the standard deviation of the statistic is known as the "Standard Error (SE)" .

SAMPLING DISTRIBUTION AND STANDARD ERROR OF STATISTIC

- SE can be regarded as a measure of precision achieved by sampling.
- SE is inversely proportional to the square root of sample size.

STANDARD ERROR OF SAMPLE MEAN

WITH REPLACEMENT

$$SE(\bar{x}) = \frac{\sigma}{\sqrt{n}} \text{ for SRS WR}$$

WITHOUT REPLACEMENT

$$SE(\bar{x}) = \frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}} \quad \text{for SRS WOR}$$

Where ,
σ **= Population SD , N = Population Size , n = sample size**

$\sqrt{\frac{N-n}{N-1}}$ **= finite population correction (fpc) or finite population multiplier**

STANDARD ERROR OF PROPORTION

WITH REPLACEMENT

$$SE(p) = \sqrt{\frac{Pq}{n}} \quad \text{for SRS WR}$$

WITHOUT REPLACEMENT

$$SE(p) = \sqrt{\frac{Pq}{n}}\sqrt{\frac{N-n}{N-1}} \quad \text{for SRS WOR}$$

Where ,

N = Population Size , n = sample size , p + q = 1

$\sqrt{\frac{N-n}{N-1}}$ = finite population correction (fpc) or finite population multiplier

may be ignored as it tends to 1 if the sample size (n) is very large or the population under consideration is infinite when the parameters are unknown, .

Example: A population comprises the following units: a, b, c, d, e. Draw all possible samples of size three without replacement.

Solution Since in this case, sample size (n) = 3 and population size (N) = 5. the total number of possible samples without replacement = ${}^{5}c_{3} = 10$

These are abc, abd, abe, acd, ace, ade, bcd, bce,bde,cde.

Example: A population comprises 3 member 1, 5, 3. Draw all possible samples of size two

(i) with replacement

(ii) without replacement

Find the sampling distribution of sample mean in both cases.

Solution (i) With replacement :- Since n = 2 and N = 3, the total number of possible samples of size 2 with replacement = $3^2 = 9$.

All possible samples of size 2 from a population comprising 3 units under WR scheme

Serial No.	Sample of size 2 with replacement	Sample mean ($\bar{x}$)
1	1, 1	1
2	1, 5	3
3	1, 3	2
4	5, 1	3
5	5, 5	5
6	5, 3	4
7	3, 1	2
8	3, 5	4
9	3, 3	3

Sampling distribution of sample mean

$\bar{x}$	1	2	3	4	5	Total
P	1 / 9	2 / 9	3 / 9	2 / 9	1 / 9	1

(ii) without replacement: As N = 3 and n = 2, the total number of possible samples without replacement = ${}^{N}C_{2} = {}^{3}C_{2} = 3$.

Possible samples of size 2 from a population of 3 units under WOR scheme

Serial No	Sample of size 2 without replacement	Sample mean ($\bar{x}$)
1	1 , 3	2
2	1 , 5	3
3	3 , 5	4

Sampling distribution of mean

:	2	3	4	Total
P:	1 / 3	1/3	1/3	1

TYPES OF SAMPLING

PROBABILITY SAMPLING

NON - PROBABILITY SAMPLING

MIXED SAMPLING

PROBABILITY SAMPLING

- In the Probability sampling there is always a fixed, pre assigned probability for each member of the population to be a part of the sample taken from that population

- Some important probability sampling other than simple random sampling are stratified sampling, Multi Stage sampling, Multi Phase Sampling, Cluster Sampling and so on.

SIMPLE RANDOM SAMPLING

- When the units are selected independent of each other in such a way that each unit belonging to the population has an equal chance of being a part of the sample, the sampling is known as Simple random sampling or just random sampling.

SIMPLE RANDOM SAMPLING

- If the units are drawn one by one and each unit after selection is returned to the population before the next unit is being drawn so that the composition of the original population remains unchanged at any stage of the sampling then the sampling procedure is known as **Simple Random Sampling with replacement.**

- If, however, once the units selected from the population one by one are never returned to the population before the next drawing is made, then the sampling is known as **Simple Random Sampling without replacement**

SIMPLE RANDOM SAMPLING

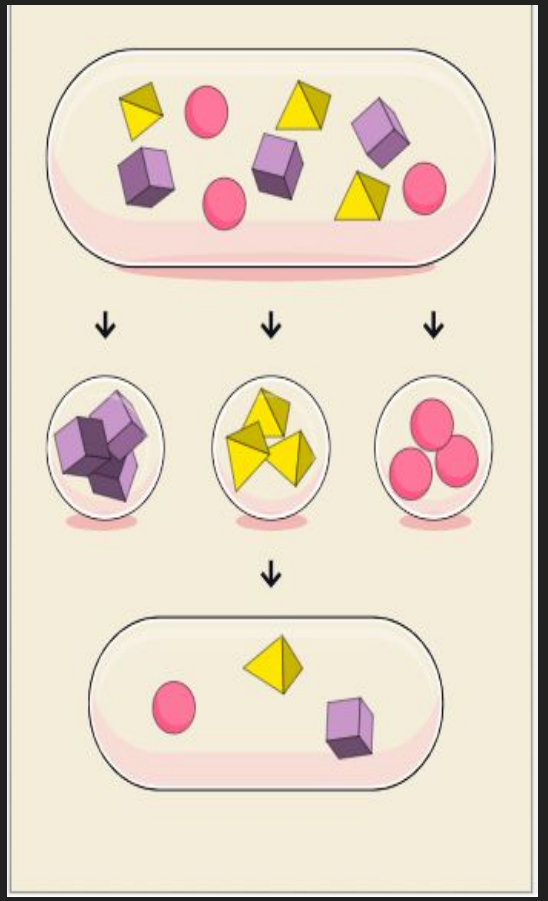
- Simple random sampling is a very simple and effective method of drawing samples provided
- (i) the population is not very large
- (ii) the sample size is not very small and
- (iii) the population under consideration is not heterogeneous i.e. there is not much variability among the members forming the population.
- Simple random sampling is completely free from Sampler's biases.

STRATIFIED SAMPLING

- In this method , the universe or the entire population is divided into a number of groups or strata and then certain number of items are taken from each group at random .

- Its basic purpose is to ensure that all the characteristics of a heterogeneous population are adequately represented in the sample .

- It helps in reduction of variability and thereby an increase in precision.



STRATIFIED SAMPLING

- There are two types of allocation of sample size.
- When there is prior information that there is not much variation between the strata variances. We consider "Proportional allocation" or "Bowely's allocation where the sample sizes for different strata are taken as proportional to the population sizes.
- When the strata-variances differ significantly among themselves, we take recourse to "Neyman's allocation" where sample size vary jointly with population size and population standard deviation

STRATIFIED SAMPLING

- **The purpose of stratified sampling are**
- (i) to make representation of all the sub populations
- (ii) to provide an estimate of parameter not only for all the strata but also and overall estimate
- (iii) reduction of variability and thereby an increase in precision.
- **Stratified sampling not advisable if**
- (i) population is not large
- (ii) some prior information is not available
- (iii) there is not much heterogeneity among the units of population

MULTISTAGE SAMPLING

- In this type of complicated sampling, the population is supposed to compose of first stage sampling units, each of which in its turn is supposed to compose of second stage sampling units, each of which again in its turn is supposed to compose of third stage sampling units and so on till we reach the ultimate sampling unit.

MULTISTAGE SAMPLING

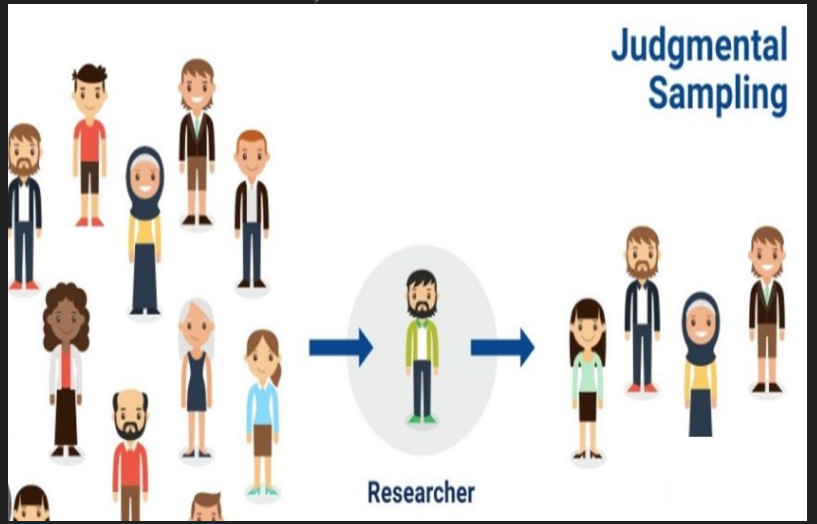
- The coverage in case of multistage sampling is quite large.
- It also saves computational labour and is cost-effective.
- It adds flexibility into the sampling process which is lacking in other sampling schemes.
- However, compared to stratified sampling, multistage sampling is likely to be less accurate.

NON - PROBABILITY SAMPLING

- In non- probability sampling , no probability attached to the member of the population and as such it is based entirely on the judgement of the sampler.
- Non-probability sampling is also known as Purposive or Judgemental Sampling

PURPOSIVE OR JUDGEMENTAL SAMPLING

- This type of sampling is dependent solely on the discretion of the sampler and he applies his own judgement based on his belief, prejudice, whims and interest to select the sample.
- Since this type of sampling is non-probabilistic, it is purely subjective and, as such, varies from person to person.
- No statistical hypothesis can be tested on the basis of a purposive sampling

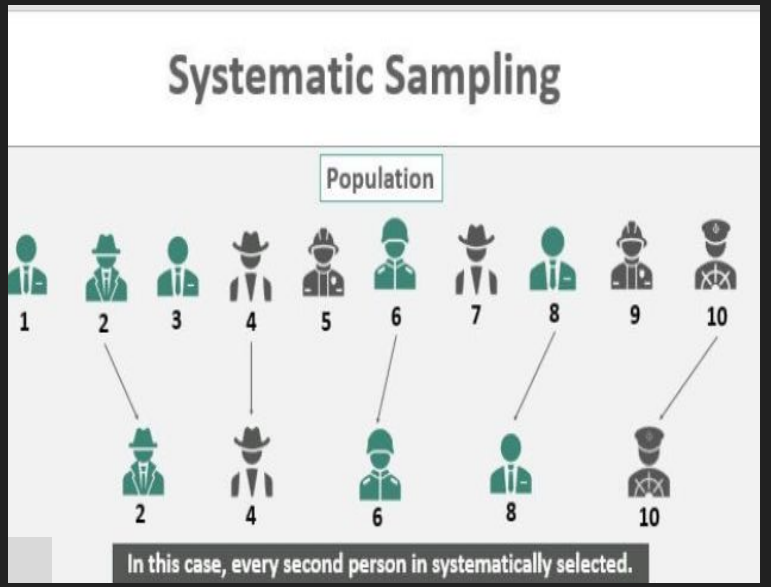


MIXED SAMPLING

- Mixed sampling is based partly on some probabilistic law and partly on some pre decided rule.
- Systematic sampling belongs to this category.

SYSTEMATIC SAMPLING

- It refers to a sampling scheme where the units constituting the sample are selected at regular interval after selecting the very first unit at random i.e., with equal probability.
- Systematic sampling is partly probability sampling in the sense that the first unit of the systematic sample is selected probabilistically and partly non- probability sampling in the sense that the remaining units of the sample are selected according to a fixed rule which is non-probabilistic in nature.



SYSTEMATIC SAMPLING

- If the population size N is a multiple of the sample size n i.e. **N = nk**, for a positive integer k which must be less than n, then the systematic sampling comprises selecting one of the first k units at random, usually by using random sampling number and thereby selecting every k^{th} unit till the complete, adequate and updated sampling frame comprising all the members of the population is exhausted. This type of systematic sampling is known as "***linear systematic sampling***". K is known as "***sample interval***".

SYSTEMATIC SAMPLING

- However, if N is not a multiple of n, then we may write $N = nk + p$, $p < k$ and as before, we select the first unit from 1 to k by using random sampling number and thereafter selecting every kth unit in a cyclic order till we get the sample of the required size n. This type of systematic sampling is known as "***circular systematic sampling***."

EXERCISE- Set (A)

CA FOUNDATION

CODE : SS12

Shivani
Sharma

Answer the following questions. Each question carries one mark.

Que. 1 Sampling can be described as a statistical procedure

(a) To infer about the unknown universe from a knowledge of any sample

(b) To infer about the known universe from a knowledge of a sample drawn from it

(c) To infer about the unknown universe from a knowledge of a random sample drawn from it

(d) Both (a) and (b).

MATHS

CA FOUNDATION

CODE : SS12

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Answer the following questions. Each question carries one mark.

Que. 2 The Law of Statistical Regularity says that

(a) Sample drawn from the population under discussion possesses the characteristics of the population

(b) A large sample drawn at random from the population would possess the characteristics of the population

(c) A large sample drawn at random from the population would possess the characteristics of the population on an average

(d) An optimum level of efficiency can be attained at a minimum cost.

MATHS

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Answer the following questions. Each question carries one mark.

Que. 3 A sample survey is prone to

(a) Sampling errors

(b) Non-sampling errors

(c) Either (a) or (b)

(d) Both (a) and (b)

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Answer the following questions. Each question carries one mark.

Que. 4 The population of roses in Salt Lake City is an example of

(a) A nite population

(b) An infinite population

(c) A hypothetical population

(d) An imaginary population.

MATHS

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Answer the following questions. Each question carries one mark.

Que. 5 Statistical decision about an unknown universe is taken on the basis of

(a) Sample observations

(b) A sampling frame

(c) Sample survey

(d) Complete enumeration

MATHS

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Answer the following questions. Each question carries one mark.

Que. 6 Random sampling implies

(a) Haphazard sampling

(b) Probability sampling

(c) Systematic sampling

(d) Sampling with the same probability for each unit.

MATHS

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CODE : SS12

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Answer the following questions. Each question carries one mark.

Que. 7 A parameter is a characteristic of

(a) Population

(b) Sample

(c) Both (a) and (b)

(d) (a) or (b)

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Answer the following questions. Each question carries one mark.

Que. 8 A statistic is

(a) A function of sample observations

(b) A function of population units

(c) A characteristic of a population

(d) A part of a population.

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Answer the following questions. Each question carries one mark.

Que. 9 Sampling Fluctuations may be described as

(a) The variation in the values of a statistic

(b) The variation in the values of a sample

(c) The differences in the values of a parameter

(d) The variation in the values of observations.

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Answer the following questions. Each question carries one mark.

Que. 10 The sampling distribution is

(a) The distribution of sample observations

(b) The distribution of random samples

(c) The distribution of a parameter

(d) The probability distribution of a statistic.

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Answer the following questions. Each question carries one mark.

Que. 11 Standard error can be described as

(a) The error committed in sampling

(b) The error committed in sample survey

(c) The error committed in estimating a parameter

(d) Standard deviation of a statistic.

MATHS

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Answer the following questions. Each question carries one mark.

Que. 12 A measure of precision obtained by sampling is given by

(a) Standard error

(b) Sampling fluctuation

(c) Sampling distribution

(d) Expectation.

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Answer the following questions. Each question carries one mark.

Que. 13 As the sample size increases, standard error

(a) Increases

(b) Decreases

(c) Remains constant

(d) Decreases proportionally.

MATHS

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Answer the following questions. Each question carries one mark.

Que. 14 If from a population with 25 members, a random sample without replacement of 2 members is taken, the number of all such samples is

(a) 300

(b) 625

(c) 50

(d) 600

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Answer the following questions. Each question carries one mark.

Que. 15 A population comprises 5 members. The number of all possible samples of size 2 that can be drawn from it with replacement is

(a) 100

(b) 15

(c) 125

(d) 25

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Answer the following questions. Each question carries one mark.

Que. 16 Simple random sampling is very effective if

(a) The population is not very large

(b) The population is not much heterogeneous

(c) The population is partitioned into several sections.

(d) Both (a) and (b)

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Answer the following questions. Each question carries one mark.

Que. 17 Simple random sampling is

(a) A probabilistic sampling

(b) A non- probabilistic sampling

(c) A mixed sampling

(d) Both (b) and (c).

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Answer the following questions. Each question carries one mark.

Que. 18 According to Neyman's allocation, in stratified sampling

(a) Sample size is proportional to the population size

(b) Sample size is proportional to the sample SD

(c) Sample size is proportional to the sample variance

(d) Population size is proportional to the sample variance.

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Answer the following questions. Each question carries one mark.

Que. 19 Which sampling provides separate estimates for population means for different segments and also an over all estimate?

(a) Multistage sampling

(b) Stratified sampling

(c) Simple random sampling

(d) Systematic sampling

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Answer the following questions. Each question carries one mark.

Que. 20 Which sampling adds flexibility to the sampling process?

(a) Simple random sampling

(b) Multistage sampling

(c) Stratified sampling

(d) Systematic sampling

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Answer the following questions. Each question carries one mark.

Que. 21 Which sampling is affected most if the sampling frame contains an undetected periodicity?

(a) Simple random sampling

(b) Stratified sampling

(c) Multistage sampling

(d) Systematic sampling

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Answer the following questions. Each question carries one mark.

Que. 22 Which sampling is subjected to the discretion of the sampler?

(a) Systematic sampling

(b) Simple random sampling

(c) Purposive sampling

(d) Quota sampling.

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Answer the following questions. Each question carries one mark.

Que. 23 If a random sample of size 2 with replacement is taken from the population containing the units 3,6 and 1, then the samples would be

(a) (3, 6),(3, 1),(6, 1)

(b) (3, 3),(6, 6),(1, 1)

(c) (3, 3),(3, 6),(3, 1),(6, 6),(6, 3),(6, 1),(1, 1),(1, 3),(1, 6)

(d) (1, 1),(1, 3),(1, 6),(6, 1),(6, 2),(6, 3),(6, 6),(1, 6),(1, 1)

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Answer the following questions. Each question carries one mark.

Que. 24 If a random sample of size two is taken without replacement from a population containing the units a,b,c and d then the possible samples are

(a) (a, b),(a, c),(a, d)

(b) (a, b),(b, c), (c, d)

(c) (a, b), (b, a), (a, c),(c,a), (a, d), (d, a)

(d) (a, b), (a, c), (a, d), (b, c), (b, d), (c,d)

MATHS